Satellite Field of View

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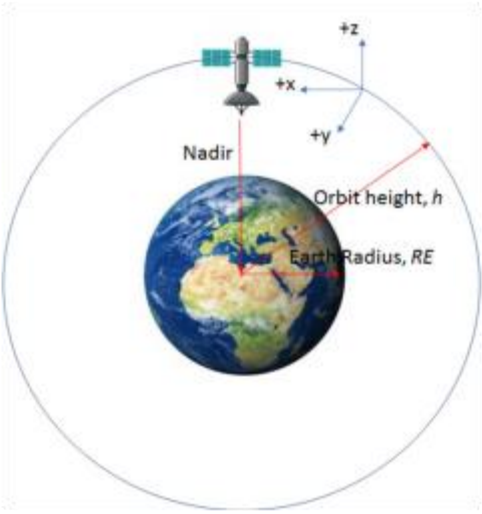
This article explains how to derive the Field of View (footprint) for a simple satellite-mounted antenna, for typical Low-Earth Orbiting (LEO) satellites for two cases:

- **Nadir pointing** - that is when the antenna's beam centre (called the "boresight") is pointing towards the geometric centre of Earth, and for a simple antenna, results in a nearly circular coverage area projected onto the Earth.
- **Off-Nadir pointing** - that is when the antenna's boresight is offset from the Nadir direction either in 2 dimensions or 3 dimensions. We shall only consider the simple case of 2-dimensions. Off-Nadir pointing results in an Elliptical coverage area projected onto the Earth.

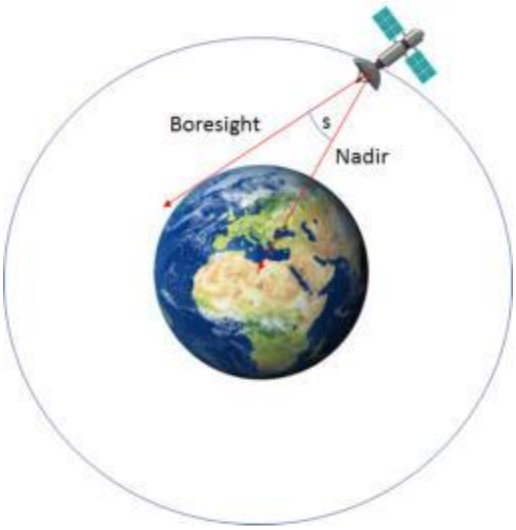
Another term often used is "illumination" meaning the Field of View (FoV) projected onto the Earth's surface. The antenna's beam can be thought of as being similar to a torch beam. As with a torch pointing directly at the ground, we observe a circular illumination. When the torch is tilted at an angle such that its beam intersects with the ground, we observe elliptical illumination.

The geometric FoV (described in this article) and the actual FoV may be significantly different due to effects such as refraction and reflection e.g. from the ionosphere, or even the Earth's surface itself. But leaving aside such effects, the geometric analysis is the same for a wide range of applications such as radio links (both reception and transmission), radiometric sensors and of course cameras. For the purposes of this article we shall consider the sensor to be a narrow-beam radio antenna.

Let the satellite's direction of motion be $+x$ (its velocity vector), Nadir is the $-z$ unit vector, and the $+y$ unit vector pointing out of the page as shown below. The satellite's orbital height is h and the Earth's radius (approximately 6372 km) is RE. In the figure below, the antenna is Nadir pointing. The point on the Earth's surface directly below the satellite is called the Sub-Satellite Point (SSP).



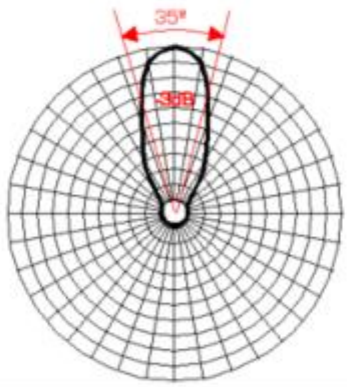
When the satellite's sensor (e.g. an antenna) is positioned with an off-Nadir angle, the antenna is slewed by an angle *s* relative to Nadir, in the *x-z* plane as shown below.



The slewing of the antenna may be achieved by means of a fixed or movable orientation of the antenna itself relative to the spacecraft or, (especially in the case of very small satellites), by moving the entire satellite so that it's antenna is orientated in the desired direction.

Antenna Beam Definition

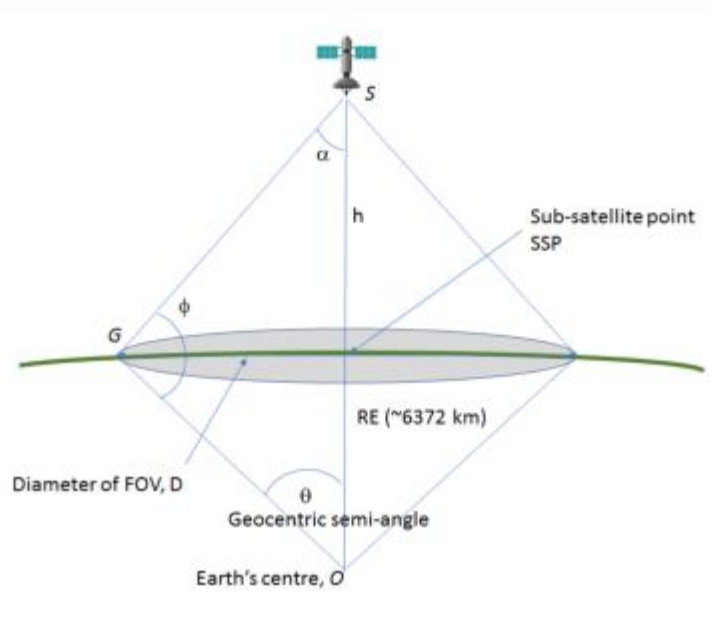
First, we need to define the antenna beam. Antennas used for command, control, telemetry or as a payload sensor are usually directional. An antenna's gain depends upon its directivity and its efficiency. The polar plot, like that shown below is used to plot the antenna's gain with respect to angle from the "boresight", usually taken to be 0 degrees, and often (but by no means always) the point of highest gain. The gain is expressed in dB relative to an isotropic antenna (dBi). The gain plot is usually normalised with respect to the maximum gain. The half-power beam width (HPBW) is an important measure for an antenna and fundamental to the calculation of coverage on the Earth.



As shown above, the HPBW is the angle from where the gain, relative to maximum drops by 3dB, either side of the boresight. In the above example, the HPBW is 35 degrees. The antenna half-angle, α is also defined. This is α = HPBW/2 i.e. 17.5 degrees in the figure above.

Nadir-pointing Field of View Geometry

First, we consider the simple case of a narrow-beam antenna pointing directly at the Earth's centre at a height *h* above the Earth's surface i.e. the antenna's boresight is aligned with the satellite's Nadir axis and angle *s*=0 degrees. The figure below illustrates the geometry and is not drawn to scale.



The satellite is orbiting at a height *h* above the Earth's surface and is therefore a total distance of *RE+h* from the Earth's centre, *O*. For the Nadir-pointing case, a triangle *OGS* is formed with angles of antenna half-angle *a*, the enclosed angle ϕ and the Geocentric semi-angle θ. *G* is a point on the ground. We want to calculate the diameter of the Field of View (FoV), *D* for a given height *h* and antenna half-angle, α. We can write,

$$\frac{\sin(\alpha)}{R_E} = \frac{\sin(\varphi)}{R_E + h}$$

therefore,

$$\sin(\varphi) = \frac{\sin(\alpha)}{R_E} \cdot (R_E + h)$$

Since,

$$\theta + \varphi + \alpha = \pi$$
,

substituting for ϕ,

$$\sin(\varphi) = \sin(\pi - \theta - \alpha) = \sin(\theta + \alpha)$$

then to derive the geocentric semi-angle θ, we can write,

$$\theta = sin^{-1}\left(sin(\alpha) \cdot \frac{R_E + h}{R_E}\right) - \alpha$$

Now, (working in radians of course) the diameter of the FoV, *D* is simply given by,

$$D = 2\theta R_E$$

For example, if α = 17.5 degrees, *h* = 550km and we take RE to be 6372km, θ = 0.0273 radians, and *D* = 348km. This represents the maximum coverage diameter where the ground point *G* is at the edge of the FoV. The angle of elevation ε at *G* is derived as follows:

Taking the elevation ε as the angle between the tangent at *G* (i.e. a line at a right-angle to *OG*) and the satellite,

$$\varepsilon = \varphi - \frac{\pi}{2}$$

Substituting for ϕ,

$$\pi = \theta + \varepsilon + \frac{\pi}{2} + \alpha$$

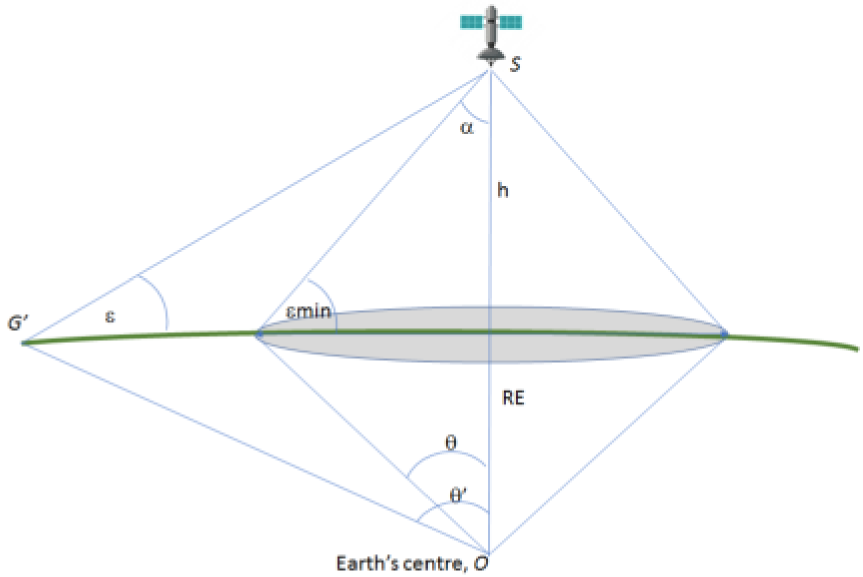
therefore,

$$\varepsilon = \frac{\pi}{2} - \theta - \alpha$$

As the point *G* is moved closer towards the sub-satellite point, SSP at the centre of the FoV, the angle of elevation ε will increase until the point where the satellite is directly overhead *G* and ε becomes 90 degrees. Note, ε produced above is the *minimum* usable elevation, ε_{min}.

Minimum Usable Elevation

Consider a ground station at G' that is outside the FoV of the satellite, as shown below.



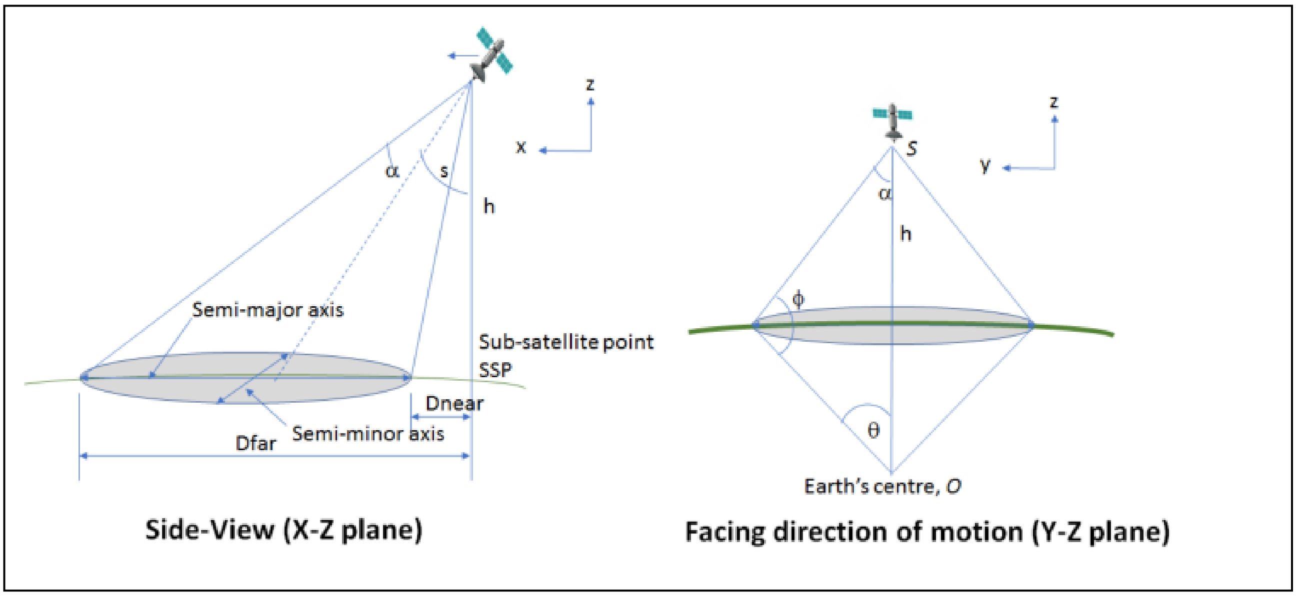
The satellite is visible with a line-of-sight path from *G'* with an elevation angle ε but the point G' lies outside the FoV. As the satellite moves towards *G'*, ε increases. Once ε reaches the minimum usable elevation εmin, the grounstation falls within the FoV. The relationship between minimum elevation and FoV is especially important for space-ground communications (e.g. Telemetry and Telecommand) because the grounstation will have a minimum usable elevation for the site which is determined by factors such as terrain, atmospheric conditions and local noise and interference. The contact (or pass) time that the ground station can offer depends upon the minimum usable elevation of the satellite's communications antenna *and* the minimum useable elevation for the ground station. The ground station and satellite beams must intersect for there to be communication. The nature of the orbit has a significant effect on the pass time. If the satellite does not fly over the ground station, but just comes over the horizon at a low angle of elevation then dips below the horizon again, the pass may not be useable unless the ground station antenna has a very low minimum usable elevation.

It is obvious that wide beam-width antennas are better suited to communication with a ground-station because its εmin is lower. Where the ground station offers a low minimum angle of elevation e.g. 5-10 degrees and the satellite's communication antenna's half-power beam width is also wide, the pass time can be increased significantly.

For payload sensors making observations of the Earth e.g. a camera, Synthetic Aperture Radar (SAR), ship detection by AIS, or other similar applications, the beam width may be narrow, or even steerable. A Nadir-pointing sensor will have a FoV centred on the Sub-satellite point (SSP) and may detect features both in front of the satellite and to the rear as it passes over the region of interest.

Off-Nadir Pointing Field of View Geometry

If the antenna is offset from the Nadir-pointing vector by an angle *s* in the *X-Z* plane, so that the boresight of the antenna points either forwards or backwards instead of vertically towards the Earth's centre, it is said to be off-nadir pointing as shown below. The figure shows the side-view (*X-Z* plane) and the vertical profile viewed from the direction of motion (i.e. the satellite is travelling out of the page towards us) in the *Y-Z* plane.



The FoV now projected onto the Earth's surface is an ellipse rather than a circle as is the case for the Nadir-pointing beam. Such a FoV may be referred to as a Swath, a term usually reserved for Earth Observation sensors e.g. SAR. The analysis we present here is restricted to tilt in the *X-Z* plane but many missions may rotate the beam in the *X-Y* plane (i.e. to "look" to the side from the satellite's perspective) so that the FoV is projected to the side of the satellite's ground track.

The ellipse has a long dimension, the Semi-major axis and a short dimension, the Semi-minor axis. If the antenna is tilted in the *X-Z* plane as shown, the geometry of the Semi-minor axis in the Y-Z plane is the same as that presented for the Nadir-pointing FoV. We make the assumption that in three dimensions the antenna beam pattern is a volume of revolution so that its half-power beam-width in the *X-Y* plane is the same as that in the *X-Z* plane.

The Field of View analysis for the off-Nadir pointing case is similar to that for the Nadir-pointing case. The elliptical FoV now has two limbs: the near limb, closest to the sub-satellite point and the far limb, furthest away from the sub-satellite point. The antenna boresight is offset from the Nadir axis by an angle *s*. Using the trigonometry presented for the Nadir-pointing analysis, we derive two geocentric semi-angles, θN and θF enabling the near and far distances D_{NEAR} and D_{FAR}

respectively to be derived. The Semi-major axis of the ellipse, $D_{SEMI\text{-}MAJOR}$ is then given by D_{FAR} - D_{NEAR}.

By substituting, $\alpha \longrightarrow s - \alpha$ for the near limb, and $\alpha \longrightarrow s + \alpha$ for the far limb,

$$\theta_{NEAR} = sin^{-1}\left(sin(s-\alpha) \cdot \frac{R_E + h}{R_E}\right) - (s - \alpha)$$

$$D_{NEAR} = \theta_{NEAR} R_E$$

$$\theta_{FAR} = sin^{-1}\left(sin(s+\alpha) \cdot \frac{R_E + h}{R_E}\right) - (s + \alpha)$$

$$D_{FAR} = \theta_{FAR} R_E$$

$$D_{SEMI-MAJOR} = D_{FAR} - D_{NEAR}$$

Note that the antenna half-angle α is offset by angle *s*.

Likewise, the elevation of the satellite at the near ground point G_{NEAR} and the far ground point G_{FAR} is given by,

$$\varepsilon_{NEAR} = \frac{\pi}{2} - \theta_{NEAR} - (s - \alpha)$$

and,

$$\varepsilon_{FAR} = \frac{\pi}{2} - \theta_{FAR} - (s + \alpha)$$

Summary

We have presented a simple geometric analysis to derive the Nadir-pointing Field of View for a beam antenna on a satellite, and using a similar analysis, the Off-Nadir Field of View. The off-Nadir analysis considered two limbs - the near limb, and the far limb. For each limb the Nadir-pointing analysis was applied to derive an expression for the Semi-major axis of the elliptical FoV.

In most cases, the Nadir-pointing FoV is circular, whereas the off-Nadir FoV is an ellipse. This 2-dimensional analysis considered tilt in the *X-Z* plane (*X* is direction of satellite motion; *Z* is the Nadir-Zenith axis). However, the analysis can equally be applied to a “side-looking” FoV in the *Y-Z* plane. A 3-dimensional analysis is of course significantly more complex compare to 2-

dimensions, but not inherently difficult using spherical geometry instead of plane geometry.

The minimum usable elevation was derived, which is important in analysing communications coverage and therefore pass duration for a space-ground communications link. It was shown that although a satellite may be visible from the ground, the communications path may not be open due to the beam-width of the communications antennas.